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PROGRESSION FOR DEVELOPING UNDERSTANDING OF VARIABLE ACROSS GRADES 3-7:

This curricular progression is intended to develop algebra understanding through a study of variable notation. Variables are linguistic tools that allow us to express mathematical ideas in succinct ways through symbolic notation (Blanton et al., 2011).

SUMMARY OF DEVELOPMENT OF IDEAS ACROSS GRADES 3-7:

The following is a proposed curricular progression for the development of an understanding of variable in grades 3-7. The progression is based on what we know from research that children can reasonably accomplish. This content is integrated into the curricular progressions of Functional Thinking; Generalized Arithmetic; and Equality, Expressions, Equations and Inequalities, rather than existing as stand-alone content. In what follows, we describe how the concepts associated with variable are addressed within each of these Big Ideas across grades 3-7. Note that variables refer only to letters (literal symbols) and any symbols in the progressions are indicated by literal symbols (i.e., no use of non-literal symbols).

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Third Grade:

In 3rd grade, students begin to explore the fundamental concept of variable. In particular, they begin to use variables to express arithmetic generalizations, simple algebraic expressions, equations, inequalities, and simple functional relationships. In these contexts, they begin their study of variable as a fixed but unknown quantity, a generalized number, or a varying, unknown quantity. They explore the meaning of repeated variables or different variables in an algebraic expression, equation, inequality, or function rule. They also begin to develop an understanding of variable as the measure or amount of an object rather than the object itself and to interpret the meaning of a variable within a problem context.

GA[1]

- use variables to represent arithmetic generalizations
- examine the meaning of a repeated variable in an equation (e.g., $a - a = 0$)
- examine the meaning of different variables in an equation (e.g., $a + b = b + a$)

EEEI

- use variables to represent a fixed but unknown quantity or a varying quantity
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- understand the meaning of a repeated variable or different variables in an expression, equation or inequality
- use variables when writing algebraic expressions, equations, and inequalities

FT

- use a variable to represent a varying, unknown quantity
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within the problem context (e.g., understand that '*x*' represents the number of pieces of string)
- describe a function rule using variables

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[1] GA – Generalized Arithmetic; EEEI – Equality, Expressions, Equations, and Inequalities; FT – Functional Thinking

- examine the meaning of different variables in a function rule

<u>Fourth Grade:</u>
In 4th grade, students refine their understanding of the concept of variable introduced in 3rd grade as they re-examine these concepts in more advanced situations. In particular, they continue to use variables to express arithmetic generalizations; simple algebraic expressions, equations, and inequalities; and simple functional relationships. They continue to examine situations in which a variable may act as a fixed but unknown quantity, a generalized number, or a varying quantity. They continue to explore the meaning of repeated variables or different variables in an algebraic expression, equation, inequality, or function rule. They also continue to explore the meaning of variable as the measure or amount of an object rather than the object itself and to interpret the meaning of a variable within a problem context.

GA
- use variables to represent arithmetic generalizations
- examine the meaning of a repeated variable in an equation (e.g., $a - a = 0$)
- examine the meaning of different variables in an equation (e.g., $a + b = b + a$)

EEEI
- identify variables to represent either a fixed, unknown or varying, unknown quantity
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within the problem context (e.g., understand that 'x' represents the number of pieces of string)
- understand the meaning of a repeated variable or different variables in an expression, equation, or inequality
- use variables when writing algebraic expressions, equations, and inequalities

FT
- use variables to represent a varying quantity
- understand that a variable represents the measure or amount of an object rather than the object itself

- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- describe a function rule using variables;
- examine the meaning of different variables in a function rule

Fifth Grade:

In 5th grade, students continue to refine their understanding of the concept of variable. In particular, they continue to use variables to represent arithmetic generalizations; algebraic expressions, equations, and inequalities; and functional relationships. They continue to explore contexts in which a variable may act as a fixed but unknown quantity, a generalized number, or a varying quantity. They reinforce their understanding of variable as the measure or amount of an object rather than the object itself and of the meaning of repeated variables or different variables in an algebraic expression, equation, inequality, or function rule. They also continue to interpret the meaning of a variable within a problem context. Finally, they extend their understanding of variable by an introduction to representations of continuous variables in coordinate graphs.

GA
- use variables to represent arithmetic generalizations
- examine the meaning of a repeated variable in the same equation (e.g., $a - a = 0$)
- examine the meaning of different variables in the same equation (e.g., $a + b = b + a$)

EEEI
- use variables to represent a fixed but unknown quantity or a varying quantity
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- understand the meaning of repeated variables or different variables in an expression, equation, or inequality
- use variables when writing algebraic expressions, equations, and inequalities

FT
- use a variable to represent a varying quantity
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- describe a function rule using variables;
- examine the meaning of different variables in a function rule
- examine graphical representations of continuous variables

Sixth Grade:

In 6th grade, students continue to refine their understanding of the concept of variable. In particular, they continue to use variables to represent arithmetic generalizations; algebraic expressions, equations, and inequalities; and functional relationships. They continue to explore contexts in which a variable may act as a fixed but unknown quantity, a generalized number, or a varying quantity. They begin to explicitly examine distinctions in different roles of variable, as either a generalized number in a pattern, a fixed but unknown quantity, or a varying quantity. They reinforce their understanding of variable as the measure or amount of an object rather than the object itself. They also continue to interpret the meaning of a variable within a problem context. Finally, they continue to examine distinctions between continuous and discrete variables by considering distinctions in how data for discrete or continuous variables are represented in coordinate graphs.

GA
- use variables to represent arithmetic generalizations
- examine the role of variable as "generalized number"

EEEI
- use variables to represent a fixed but unknown quantity or a varying quantity
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)

Variable LP   Project LEAP (Blanton & Knuth, NSF DRK-12 #1207945)

- use variables when writing algebraic expressions, equations, and inequalities
- examine role of variable as fixed but unknown quantity

FT
- use a variable to represent a varying, unknown quantity
- examine role of variable as a varying quantity (that is, that a variable in a function can take on a range of values)
- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- describe a function rule using variables;
- examine distinctions in graphical representations of continuous versus discrete variables

Seventh Grade:
In 7th grade, students continue to refine their understanding of the concept of variable. In particular, they continue to use variables to represent arithmetic generalizations; algebraic expressions, equations, and inequalities; and functional relationships. They extend their previous study of the role of variable as either a fixed but unknown quantity, a generalized number, or a varying quantity to include the role of variable as parameter. They continue to examine distinctions in the different roles of variable in these situations. They reinforce their understanding of variable as the measure or amount of an object rather than the object itself. They also continue to interpret the meaning of a variable within a problem context. Finally, they continue to examine distinctions between continuous and discrete variables by considering distinctions in how data for discrete or continuous variables are represented in coordinate graphs.

GA
- use variables to represent arithmetic generalizations
- examine role of variable as "generalized number"

EEEI
- use variable(s) to represent a fixed, unknown quantity or a varying, unknown quantity

- understand that a variable represents the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- use variables when writing algebraic expressions, equations, and inequalities
- examine role of variable as fixed but unknown quantity

FT
- use a variable to represent a varying, unknown quantity
- examine role of variable as a varying quantity (that is, that a variable in a function can take on a range of values)
- understand a variable to represent the measure or amount of an object rather than the object itself
- interpret the meaning of a variable within a problem context (e.g., understand that '*x*' represents the number of pieces of string)
- describe a function rule using variables
- examine distinctions in graphical representations of continuous versus discrete variables
- explore variable as parameter (and varying quantity) to represent a family of functions